MATH 1505
MATH FOR LIFE & SOCIAL SCIENCES

TEST 1 REVIEW

Linear Equations

3 ways to express the equation of a line:

Standard Form: $Ax + By + C = 0$ (A is generally positive)
Point-Slope Form: $y - y_0 = m(x - x_0)$ (m = slope, x_0, y_0 are a point on the line)
Slope-Intercept Form: $y = mx + b$ (m = slope, b = y-intercept)

Typically, the equation of a line is expressed in slope-intercept form whenever an a problem asks you to find a linear equation, unless otherwise specified.

Given 2 points $(x_1, y_1), (x_2, y_2)$ Slope $m = \frac{y_2 - y_1}{x_2 - x_1}$

In order to get the equation of a line, you either need:

1) 1 point and a slope **OR**
2) 2 points

Example Find the equation of the line through (-1, 6) with slope 10.

↑ ↑
x y

$y - y_0 = m(x - x_0)$ - Always start with this equation
$y - 6 = 10(x - (-1))$ – Plug in the slope and point the line must pass through
$y - 6 = 10(x + 1)$ - Simplify
$y - 6 = 10x + 10$
$y = 10x + 10 + 6$ – Solve for Y

$y = 10x + 16$ This is the final answer in slope-intercept form.
$10x - y + 16 = 0$ This is the final answer in standard form

Example Find the equation through (-2, 5), (-1, -4)

↑ ↑ ↑ ↑
x_1 y_1 x_2 y_2

Slope = $\boxed{m = \frac{y_2 - y_1}{x_2 - x_1}} = \frac{-4-5}{-1-(-2)} = \frac{-9}{1} = -9$

$y - 5 = -9(x - (-2))$
$y - 5 = -9(x + 2)$
$y - 5 = -9x - 18$
$y = -9x - 18 + 5$

$y = -9x - 13$ This is the final answer in slope-intercept form.
$9x + y + 13 = 0$ This is the final answer in standard form

15. Biologists have noticed that the **chirping rate of crickets** of a certain species is related to temperature, and the relationship appears to be very nearly linear. A cricket produces 113 chirps per minute at 70°F and 173 chirps per minute at 80°F.

(a) Find a linear equation that models the temperature T as a function of the number of chirps per minute N.

(b) What is the slope of the graph? What does it represent?

(c) If the crickets are chirping at 150 chirps per minute, estimate the temperature.

12. At the surface of the ocean, the water pressure is the same as the air pressure above the water, 15 lb/in^2. Below the surface, the water pressure increases by 4.34 lb/in^2 for every 10 ft of descent.

(a) Express the water pressure as a function of the depth below the ocean surface.

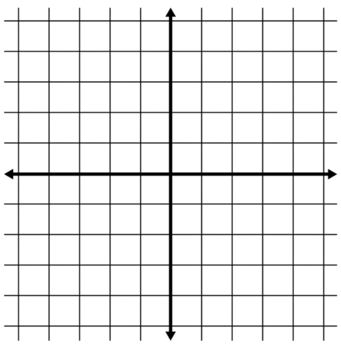
(b) At what depth is the pressure 100 lb/in^2?

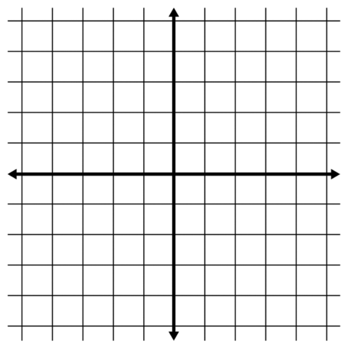
Special Linear Equations

Equation of Vertical Lines

$x = h$, where h is the x-value of where the vertical line is located.
Vertical lines have undefined slope.

Equations of Horizontal Lines
y = k, where k is the y value of where the horizontal line is located.
Horizontal lines have slope = 0.





Parallel vs Perpendicular/Normal Lines

2 lines are parallel if their slopes are equivalent
2 lines are perpendicular if the relationship between the two is $m_1 = -\frac{1}{m_2}$

Example - What is the equation of a line that passes though (3,-2) and is normal to -3x+2y+2=0

Example What is the equation of a line that passes though (3,-2) and is parallel to the line passing through (2,6) and (2,9)

Interval Notation

<u>Open Intervals (curved brackets):</u>

(a,b) can be seen as the inequality $a < x < b$

<u>Closed Intervals (square brackets):</u>

[a,b] can be seen as the inequality $a \le x \le b$

<u>Half-Open Intervals (mixed brackets):</u>

[a,b) can be seen as the inequality $a \le x < b$

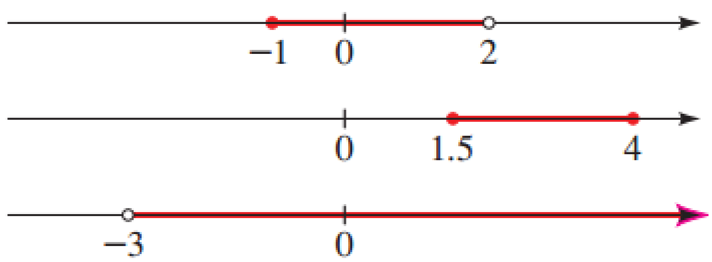
(a,b] can be seen as the inequality $a < x \le b$

Note, $\pm\infty$ is always closed by a curved bracket.

Example:

Convert the following interval graphs into interval notation.

Note, a solid circle indicates the end point is included in the interval, an hollow circle indicates the endpoint is not included in the interval.



[-1,2)

[1.5,4]

(-3,∞)

What if there are multiple answers on a number line? How do you connect them?

Use the symbol "∪ "

Ex. (-2, 0) ∪ [1.5, 4]

Absolute Value

Definition

$$|x| = \begin{cases} x & if\ x \geq 0 \\ -x & if\ x < 0 \end{cases}$$

$|x| = a$ if and only if $x = \pm a$

$|x| < a$ if and only if $-a < x < a$

$|x| > a$ if and only if $x > a$ or $x < -a$

Example Solve $|3x - 2| = -4$

Example Solve $|3x - 2| = 4$

Example Solve $|3x + 1| = 6x + 2$

Example Solve $|2x + 3| > 3$

Example Solve $|-2x + 3| \leq 1$

Equation of the Circle

$(x - x_0)^2 + (y - y_0)^2 = r^2$ r = radius of the circle (x_0, y_0) is the center of the circle

Unit Circle
The unit circle is a circle of radius 1 centered at the origin.
The equation of the unit circle is $x^2 + y^2 = 1$

Example What is the origin and radius of the circle $(x - 4)^2 + (y + 3)^2 = 9$?

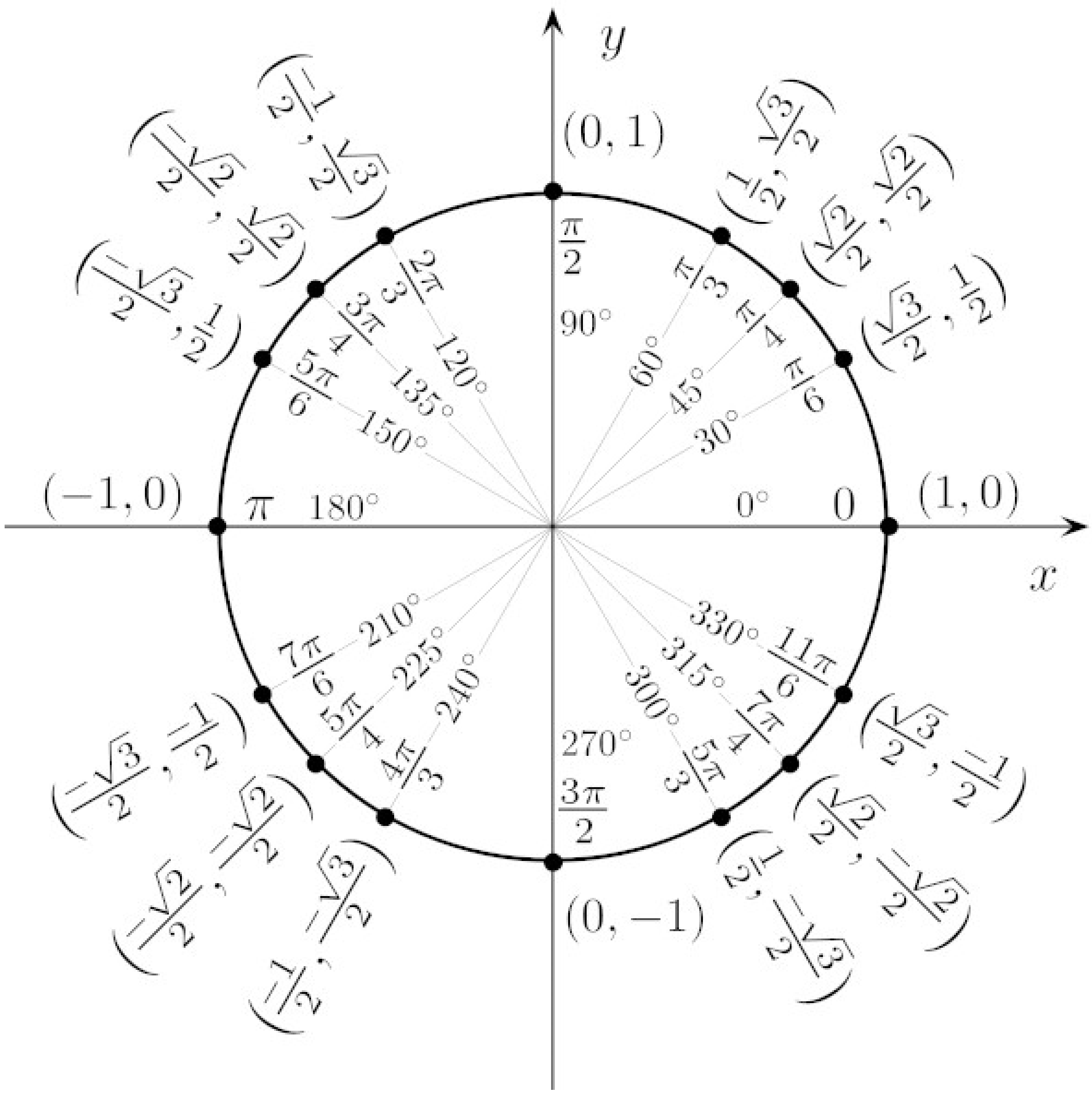
Example - What is the equation of a circle with diameter 10 centered at (2,4)?

Example - What is the equation of a circle that is centered at (1,2) and passes through (5,5)?

Example - What is the centre and radius of the circle $x^2 + y^2 + 4x - 6y = 23$

Example What is the centre and radius of the circle $x^2 + y^2 - 3x + 8y = 10$

The Unit Circle



$\cos(\theta)$ = x coordinate value

$\sin(\theta)$ = y coordinate value

$$\tan(\theta) = \frac{y}{x}$$

Note: $\frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$ These are equivalent!

Example

Solve $2sin\theta cos\theta = cos\theta$ on $[0,2\pi)$

Find the values of θ that satisfy $\tan^2 x = \tan x \quad on\ [0,2\pi)$

Find the values of θ that satisfy $|\sin x| = \sin x + \sqrt{2} \ on\ [0,2\pi)$

Domains

There are a few cases for domains that you should simply know:

1. If you have a "rational" function, the denominator cannot equal zero.
2. If you have a log or ln, the argument must be strictly greater than zero.
3. If you have a square root, the argument must be greater than or equal to zero. As above:
4. When evaluating composite functions you must also consider the domain of the second function.
   a. For example if you are finding the domain of f(g(x)) , the domain is the union of the answer as well as g(x).

For all other cases, a bit of extra leg work is required. Note: If there are multiple restrictions, find each restriction than find the overall domain.

Summary

$\sqrt{f(x)} \rightarrow f(x) \geq 0$ | $\ln(f(x)) \rightarrow f(x) > 0$ | $\frac{f(x)}{g(x)} \rightarrow g(x) \neq 0$

Example What is the domain of $y = \frac{1}{x^2+8x+12}$

Example What is the domain of $f(x) = \ln(x^2 - 9)$

Example What is the domain of $y = \frac{5x-2}{\sqrt{2x+5}}$

Example What is the domain of $y = \sqrt{\frac{x}{2+x}}$

Example – Given $f(x) = \frac{x^2+3x+2}{x+1}$

a) Determine $f(0)$

b) Determine $f(2)$

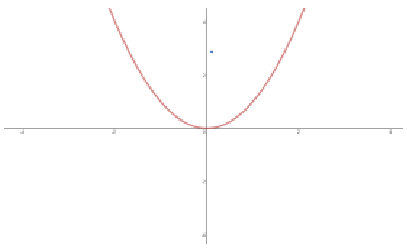
c) Determine $f(a)$

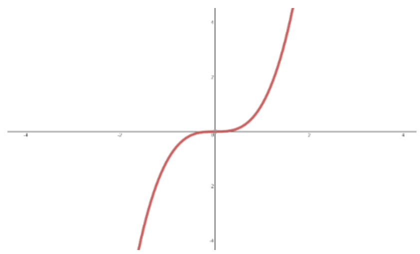
d) Determine $f(x + h)$

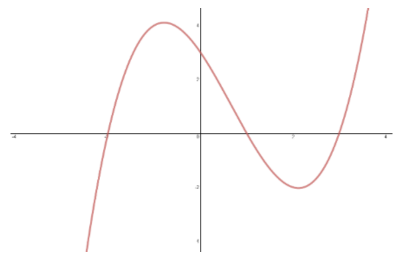
Example – Given $f(x) = \frac{x+1}{1-x}$ determine $\frac{f(x+h)-f(x)}{h}$

Odd/Even Functions

-A function is considered even if $f(x) = f(-x)$ for all x values. (Symmetric about y-axis)
-A function is considered odd if $f(x) = -f(-x)$ for all x values. (Symmetric about the origin)
-A function can be even, odd, or neither.







Example

Is $f(x) = x^2 - 1$ even, odd, or neither?

Is $f(x) = \frac{3x^3}{x^2+1}$ even, odd, or neither?

Combinations of Functions

$(f+g)(x) = f(x) + g(x)$

$(f-g)(x) = f(x) - g(x)$

$(fg)(x) = f(x) \cdot g(x)$

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} \quad g(x) \neq 0$

Examples
If $f(x) = x^2 + 2x$ and $g(x) = 1 - 2x$

Ex. 1 $(f+g)(x)$
$= f(x) + g(x)$
$= x^2 + 2x + 1 - 2x$
$= x^2 + 1$

Ex. 2 $(f-g)(x)$
$= f(x) - g(x)$
$= x^2 + 2x - (1 - 2x)$
$= x^2 + 2x - 1 + 2x$
$= x^2 + 4x + 1$

Ex. 3 $(fg)(x)$
$= f(x) \cdot g(x)$
$= (x^2 + 2x)(1 - 2x)$

Ex. 4 $\left(\frac{f}{g}\right)(x)$
$= \frac{f(x)}{g(x)}$
$= \frac{x^2+2x}{1-2x}$

Examples
If $f(x) = x^2 + 2x$ and $g(x) = \sqrt{x} - 1$

Find $\frac{f}{g}$ and it's domain

Composite Functions

$(f \circ g)(x) = f[g(x)]$ *"plug x into g(x), and plug g(x) into f(x)"*

Note: When finding domain, make sure to find domains of 'inside' function as well

Example

Given $f(x) = \sqrt{x+1}\ and\ g(x) = 2x^2 - 5x + 3$, find $(f \circ g)(x)$, $(g \circ f)(x)\ and\ (g \circ g)(x)$

Example

Given $f(x) = \frac{1}{x^2-6}\ and\ g(x) = \sqrt{x+2}$ find $(f \circ g)(x)$

Also determine the domain of the composite function.

Inverse Functions

If $f(x)$ is a one-to-one function with domain A and range B, then it's inverse function $f^{-1}(x)$ has a domain B and range A. A inverse function only exists for one-to-one functions.

NOTE: $f^{-1}(x)$ is not the same as $\frac{1}{f(x)}$, this is a common misconception at first

Finding the inverse

Once a function is determined to be one-to-one, it's inverse can be found in a few simple steps.

Steps: Interchange x and y and solve for y and then change y to $f^{-1}(x)$

Note: The domain of $f(x)$ is the range of $f^{-1}(x)$, and the range of $f(x)$ is the domain of $f^{-1}(x)$

Example

Find the inverse of $f(x) = \frac{2x+4}{x-3}$. Prove the function is one-to-one. What is the domain and range of the inverse function?

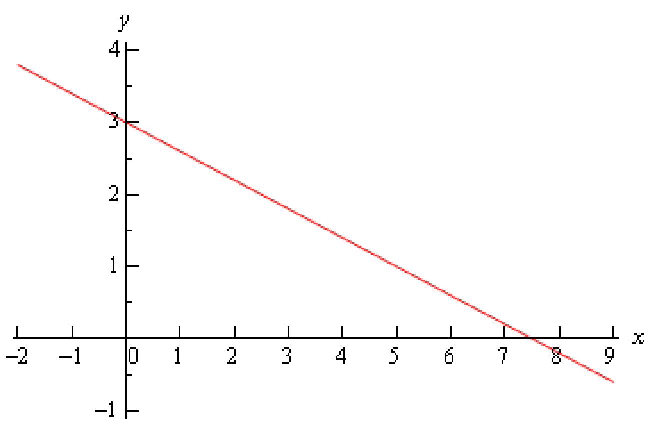
Find the inverse of $f(x) = \sqrt{3x - 2}$. What is the range of the inverse function?

Functions

A function is a relationship between an independent variable and a dependent variable such that for each value of the independent variable there exists one and only one value of the dependant variable. You are responsible for knowing the graphs of certain basic functions, their domain and ranges, how to transform functions using parameters a, b, h and k, how to find their inverses and composite functions.

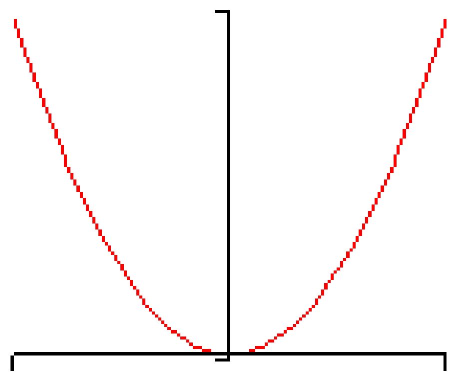
Basic Functions

The basic functions that should be known include: linear, second degree, third degree, exponential, logarithmic, absolute value, square root, and rational. The following diagrams illustrate each.



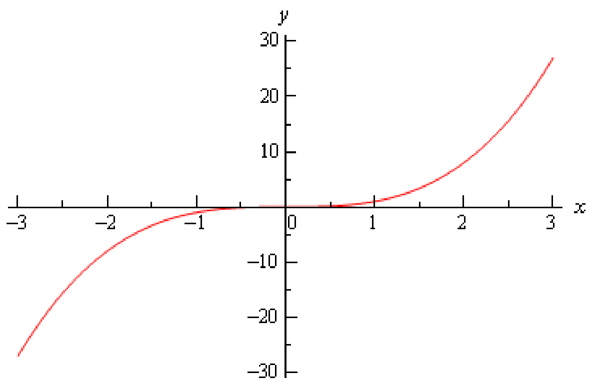
Linear Function:
y = mx + b

Domain: x ϵ IR
Range: y ϵ IR



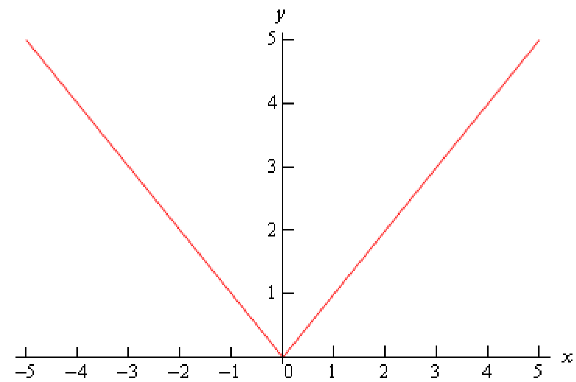
Quadratic or 2nd Degree Function:
$y = x^2$

Domain: x ϵ IR
Range: y ϵ [0, ∞) – smile



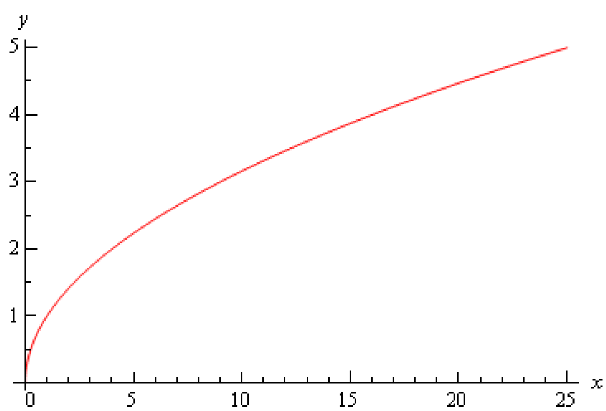
Cubic or 3rd Degree Function:
$y = x^3$

Domain: x ϵ IR
Range: y ϵ IR



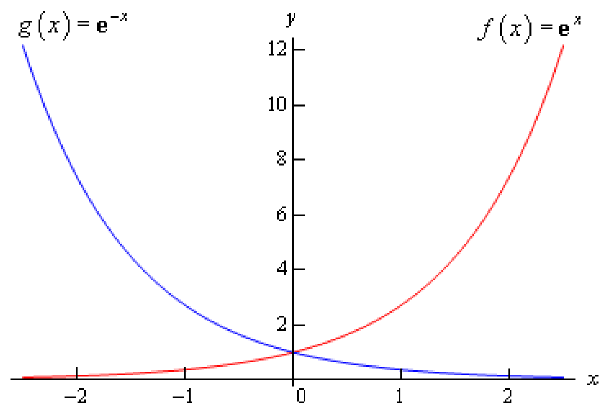
Absolute Value Function
$y = |x|$

Domain: x ϵ IR
Range: y ϵ [0, ∞)



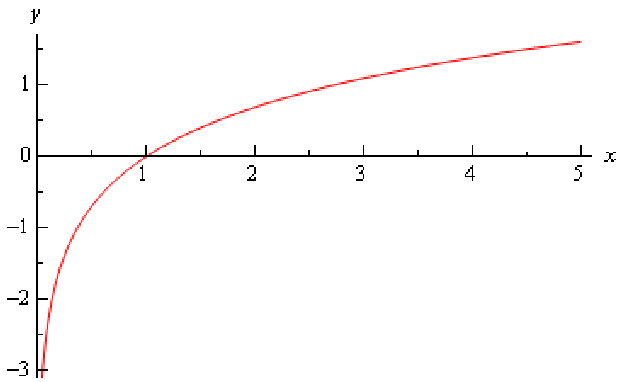
Square Root:
$y = \sqrt{x}$

Domain: x ϵ [0, ∞)
Range: y ϵ [0, ∞)



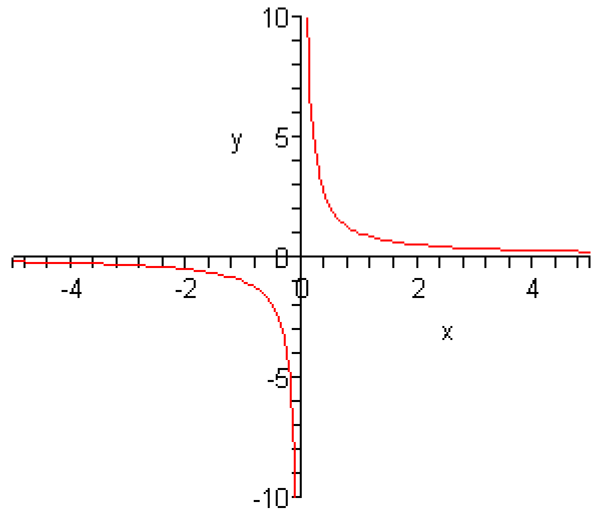
Exponential Function
$y = e^x$

Domain: x ϵ IR
Range: y ϵ (0, ∞)



Logarithmic Function:
y = logx

Domain: x ϵ (0, ∞)
Range: y ϵ (-∞, ∞)



Rational Function
y = 1/x

Domain: x ϵ IR \ {0}
Range: y ϵ IR \ {0}

Transforming functions

Any function can be transformed using the parameters a, b, h and k:

$$y = f(x) \rightarrow y = af[b(x+h)] + k$$

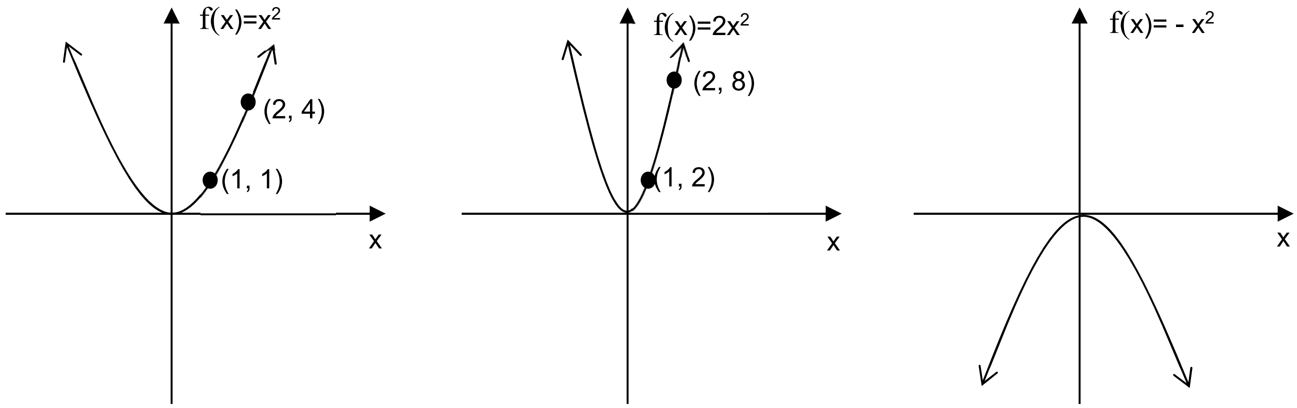
Summary

$y = f(x) + k$	$k > 0$ move graph up $k < 0$ move graph down
$y = f(x + h)$	$h > 0$ move graph left $h < 0$ move graph right
$y = af(x)$	$\|a\| > 1$ stretch in y direction $0 < \|a\| < 1$ compress in y direction Additionally, if a<0 reflect about the x-axis
$y = f(bx)$	$\|b\| > 1$ compress in x direction $0 < \|b\| < 1$ stretch in x direction Additionally, if b < 0 reflect about the y-axis

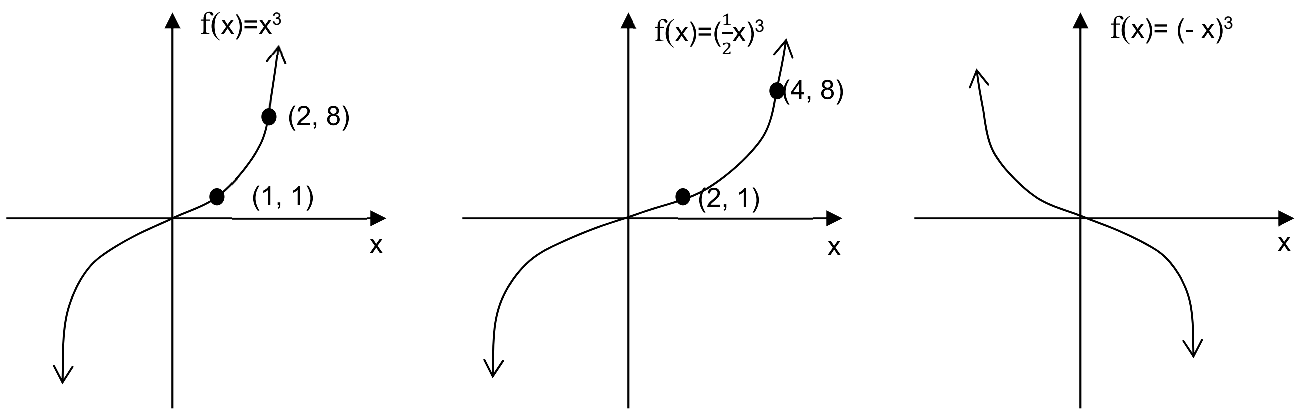
<u>Order: Reflect, Stretch/Compression, Shift</u>

When writing what factor you are **<u>stretching/compressing</u>** by, the number must be written so that it is > 1. (i.e. you don't say vertical compression by $\frac{2}{3}$, rather you would flip the number and say vertical compression by $\frac{3}{2}$).

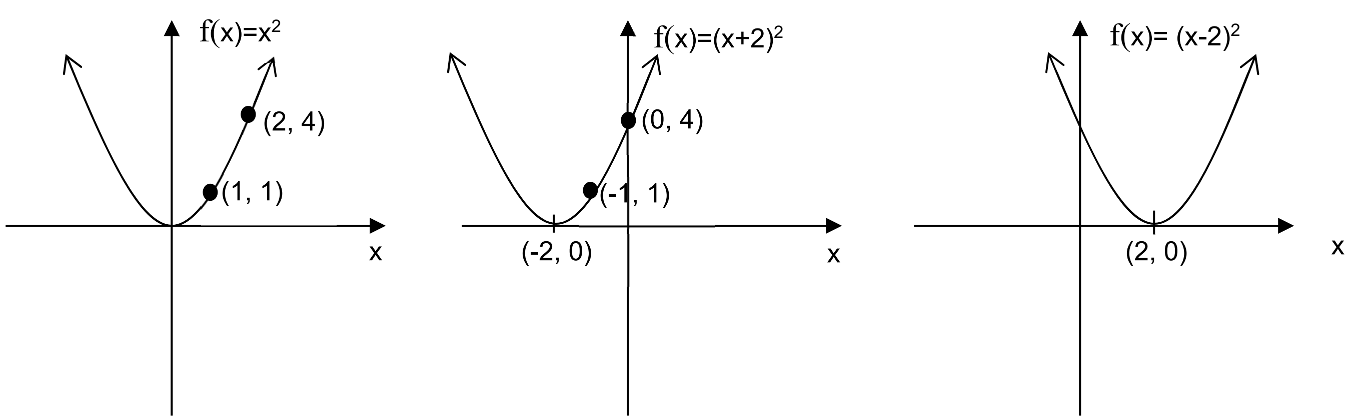
Parameter a: Vertical scale change all the y-values get multiplied by ‘a’ where as the x-values remain unchanged. If ‘a’ is negative, the function is reflected about the x-axis.



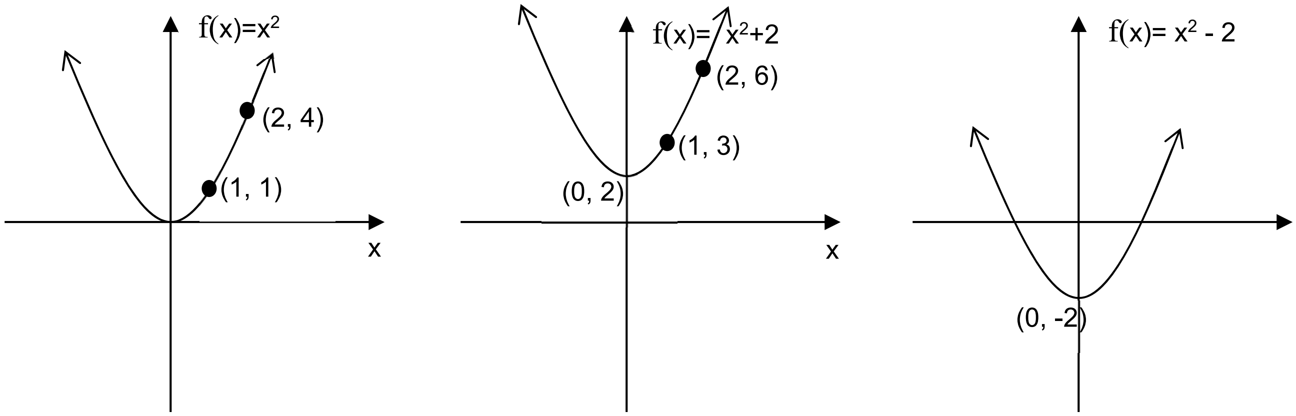
Parameter b: Horizontal scale change all the x-values get multiplied by ‘b’ where as the y- values remain unchanged. If ‘b’ is negative, the function is reflected about the y-axis



Parameter h: Horizontal translation, the function shifts left/right according to the value of h.



Parameter k: Vertical translation, the function shifts up/down according to the value of k.



Example

Describe the transformation of $f(x) = x^2$ in order to get the function $f(x) = -\frac{1}{2}(x-3)^2 + 1$

Solution

1. Reflect about x-axis to get $-x^2$
2. Compress vertically by a factor of 2 to get $-\frac{1}{2}x^2$
3. Shift to the right by 3 to get $-\frac{1}{2}(x-3)^2$
4. Shift up by 1 to get $-\frac{1}{2}(x-3)^2 + 1$

Example

Describe the transformation $f(x) = \ln(x)$ in order to get the function $f(x) = \ln(x-2) + 3$

Describe the transformation $f(x) = x^3$ in order to get the function $f(x) = -\frac{1}{4}(-2x-2)^3 - 1$

Describe the transformation $f(x) = \sin(x)$ to get the function $f(x) = -3\sin\left(x - \frac{\pi}{4}\right) - 1$

Example Sketch the graph of $f(x) = \ln(-x - 2) + 3$

Exponent Rules

You need to know these rules well. You will be using them constantly throughout the course and they will help with simplifying equations

Rule	Example
$\frac{1}{a^n} = a^{-n}$	$\frac{1}{x^{-5}} = x^5$ or $\frac{1}{x^2} = x^{-2}$
$a^m a^n = a^{m+n}$	$x^5 x^3 = x^{5+3} = x^8$
$\frac{a^m}{a^n} = a^{m-n}$	$\frac{x^{10}}{x^5} = x^{10-5} = x^5$
$(a^m)^n = a^{mn}$	$(x^6)^2 = x^{6(2)} = x^{12}$
$(ab)^m = a^m b^m$	$(xy)^3 = x^3 y^3$
$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$	$\left(\frac{x}{y}\right)^2 = \frac{x^2}{y^2}$
$\sqrt[n]{a^m} = a^{\frac{m}{n}}$	$\sqrt[4]{x^7} = x^{\frac{7}{4}}$

Note 1: This **does not** mean $(a \pm b)^n = a^n \pm b^n$
This is a VERY common mistake.

Note 2: $\sqrt[n]{a^m} = a^{\frac{m}{n}}$ This is equlvalent to $(a^m)^{\frac{1}{n}}$ or $\left(a^{\frac{1}{n}}\right)^m$

Note: For a square root $\sqrt{}$ the n-term is assumed to be 2

Logarithms

Logarithmic Form | **Exponential Form**

$y = \log_a x$ $\quad$ $x = a^y$ $\quad$ $3 = log_2 8 \leftrightarrow 8 = 2^3$

Logarithmic Properties

You need to know these rules well. You will be using them constantly throughout the course and they will help with simplifying. Get a good grasp of them now and do the associated homework problems!
Each rule is bidirectional, which means you can go left to right or right to left.

Rule	Note	Example
$log_a x^m \leftrightarrow m log_a x$	Exponent rule	$log_3(x+2)^6 \leftrightarrow 6log_3(x+2)$
$log_a x + log_a y \leftrightarrow log_a(xy)$	Sum/Product Rule	$log_3 x + log_3 y^2 \leftrightarrow log_3(xy^2)$
$log_a x - log_a y \leftrightarrow log_a\left(\frac{x}{y}\right)$	Difference/Quotient Rule	$log_3 x - log_3 y^2 \leftrightarrow log_3\left(\frac{x}{y^2}\right)$
$a^{log_a x} = x$	Cancellation Rule	$3^{log_3(x+3x^2)} = x + 3x^2$
$log_a a^x = x$	Cancellation Rule	$\log_3 3^{x+3x^2} = x + 3x^2$

Common Results
$log_a a = 1$
$log_a 1 = 0$

Commonly Used Bases
$log(x) = log_{10} x$

$\ln(x) = log_e x$

→This also called the 'natural logarithm'
→Since it's still a logarithm, all the rules above still apply

Change Of Base Formula
This formula allows you to go from base a to base b

$$log_a x = \frac{\log_b x}{\log_b a}$$

A calculator usually only has built in functions for ln (base e) and log (base 10). If you need to plug in an equation that has any other base, you have to use the change of base formula.

$$log_a x = \frac{\log x}{log a} = \frac{\ln x}{ln a}$$

Example

Simplify $2\log x - \log 3x + \log x - log x^3$

Solving Logarithmic Equations

Step 1: Bring all *x*'s to one side and numbers to the other side
Step 2: Get rid of any numbers in front of a logarithm $log_a x^m \leftrightarrow mlog_a x$
Step 3: Collapse logs on each side of the equation

$$\log_a x + log_a y \leftrightarrow log_a(xy) \quad log_a x - log_a y \leftrightarrow log_a\left(\frac{x}{y}\right)$$

Step 4: Use $log_b a = c \leftrightarrow a = b^c$ or $a^{log_a x} = x$ to get rid of logarithms and solve for x
Step 5: Check your answer. **<u>You can't have a logarithm with a negative number.</u>**

Example Solve $2\ln\left(\sqrt{10}\right) - \ln(7 - x) = \ln(x)$

Example $\log_2(x) + \log_2(x + 2) = 3$

Example Solve $log x + \log(x-1) = \log(3x+12)$

Solving Exponential Equations

Example

Solve $3^{2x+1} = 27$

$3^{2x+1} = 30$

Solve $e^{2x+3} - 2 = 0$

Solve $e^{2x} - e^{x} - 6 = 0$

Questions Involving Logarithms and Exponents

Solve $9^{\log_3 x} = 16$

Simplify $4^{\log_2 x} = \frac{25}{4}$

Example

Simplify $e^{4\ln 3}$

Example

Find the inverse of $y = \ln\,(2e^{3x} - 4)$

Application – Decay/Half Life & Growth/Doubling

	Decay or Half Life	Growth or Doubling
Base *e*	$M(t) = M_0 e^{kt}$	$M(t) = M_0 e^{kt}$
Base 2	$M(t) = M_0 2^{-t/\mathrm{h}}$	$M(t) = M_0 2^{t/d}$

- $M(t)$ is the amount remaining at time t
- M_0 is the initial amount
- t is a point in time
- k is the decay or growth rate
    - k is positive for growth or doubling questions
    - k is negative for decay or half life questions
- h is the half life [amount of time it takes the initial amount to become half the original size]
- d is the doubling time [amount of time it takes the initial amount to become double the original size]

Example

A bacterial colony has an initial population of 20 bacteria. The population doubles every 5 days.

A) Find an expression that models the growth of the bacterial population using the natural base.
B) What is the population in 15 days?
C) How long until the population is 1000?

Example

After 15 days, a radioactive substance decays to 20% of its original amount. After how many days does it decay to 60% of its original amount?

Limits [2.3, 2.4]

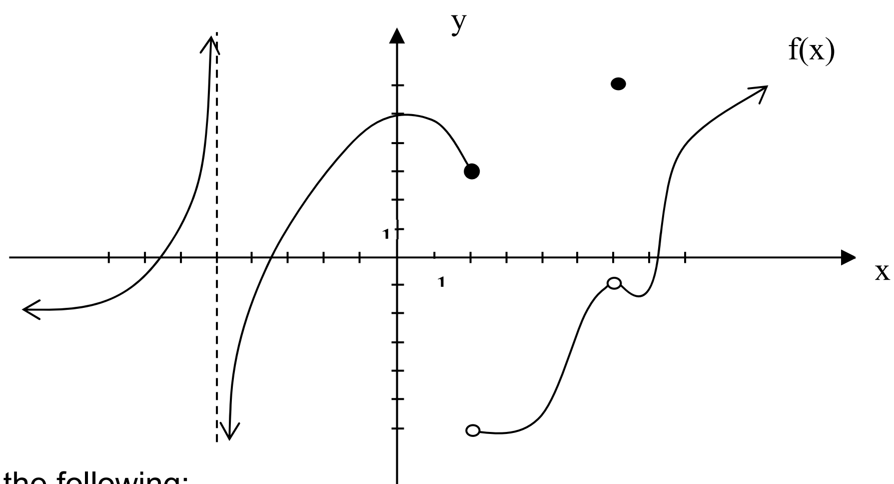
$\lim_{x \to c} f(x) = L$ "as x approaches c, $f(x)$ approaches L".

That is, what is the value of $f(x)$ as we approach x=c, but **<u>not at</u>** x=c.
The limit exists if and only if $\lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x) = L$ and only L is a finite number.
The limit does not exist if $\lim_{x \to c} f(x) = \pm\infty$

(Left sided limit) $\lim_{x \to c^-}$ $\lim_{x \to c}$ $\lim_{x \to c^+}$ (Right sided limit)

Graphical Limits [2.3]

The limit of a function as x → a ("x approaches a") is the y-value we approach as we get close to that x value. Consider the following graph.



Compute the following:

$\lim_{x \to -5^+} f(x) =$ $\lim_{x \to -5^-} f(x) =$ $\lim_{x \to -5} f(x) =$ $f(-5) =$

$\lim_{x \to 2^+} f(x) =$ $\lim_{x \to 2^-} f(x) =$ $\lim_{x \to 2} f(x) =$ $f(2) =$

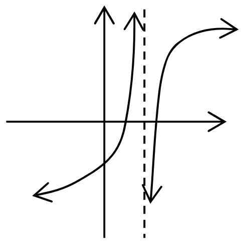
$\lim_{x \to 6^+} f(x) =$ $\lim_{x \to 6^-} f(x) =$ $\lim_{x \to 6} f(x) =$ $f(6) =$

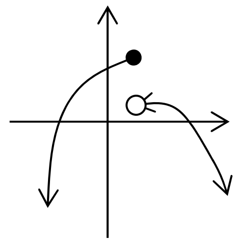
$\lim_{x \to 0^+} f(x) =$ $\lim_{x \to 0^-} f(x) =$ $\lim_{x \to 0} f(x) =$ $f(0) =$

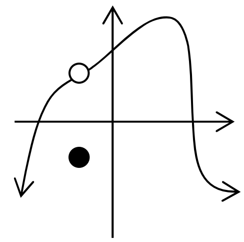
Note that if the limit from the left is not equal to the limit from the right then the limit does not exist.

Continuity [2.5]

A function is said to be continuous if you can draw it without having to lift your pencil. There are 3 types of discontinuities: (1) Infinite, (2) Jump and (3) Removable







In terms of problem solving, a function is continuous at 'a' if the following three conditions hold:

$$\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x) = f(a)$$

In other words, the ALL of the following 3 conditions MUST be true for a continuous function.

1) The function $f(x)$ must be defined at x=c
2) $\lim_{x\to c} f(x)$ exists
3) $\lim_{x\to c} f(x) = f(c)$

The following functions are continuous everywhere they are defined. That is, they are continuous on their domain.

- Polynomial Functions
- Rational Functions
- Power Functions
- Trigonometric Functions
- Exponential Functions
- Logarithmic Functions

Example For the following function, What is $\lim_{x \to 2} f(x)$? Is $f(x)$ continuous at $x = 2$? [2.3-2.5]

Given $f(x) = \begin{cases} x - 3 \; if \;\; x \neq 2 \\ 4x - 9 \; if \;\; x = 2 \end{cases}$

Example Is the following function continuous at x=1? Justify your answer. [2.3-2.5]

$$f(x) = \begin{cases} 2x^2 + 5x + 1 \; if \;\; x > 0 \\ 8e^x \; if \;\; x \leq 0 \end{cases}$$

Example Find the value of the constant a that makes the function continuous at 1. [2.3-2.5]

$$f(x) = \begin{cases} \frac{1}{x} & \text{if} \quad x \geq 1 \\ 2x + a & \text{if} \quad x < 1 \end{cases}$$

Techniques to Finding Limits

Technique 1 - Direct Substitution (Plug In) [2.3,2.4]

As the name suggests, here you simply plug in the value x approaches. For example:

$$\lim_{x\to 3}\frac{x^2-3x}{4x-2}+10x=\frac{(3)^2-3(3)}{4(3)-2}+10(3)=\frac{9-9}{12-2}+30=\frac{0}{10}+30=30$$

The question is when does plugging in fail? It fails when you obtain an indeterminate form. Common indeterminate forms include:

$$\frac{0}{0},\frac{\infty}{\infty},1^{\infty},\infty^{0},0\cdot\infty$$

Note that the following are acceptable answers:

$$\frac{0}{\#}=0,\frac{\infty}{\#}=\infty,\frac{\#}{\infty}=0,\frac{\#}{0}=\infty,\frac{0}{\infty}=0,\frac{\infty}{0}=\infty$$

here, # represents any number.

If you obtain an indeterminate form you must apply another technique. The majority of the time you will have to factor:

Technique 2: Factoring [2.4]

$$\lim_{x\to 7}\frac{x^2-49}{x^2-5x-14}=\frac{(7)^2-49}{(7)^2-5(7)-14}=\frac{49-49}{49-35-14}=\frac{0}{0}\to \textit{no good}!\to \textit{factor}:$$

$$\lim_{x\to 7}\frac{x^2-49}{x^2-5x-14}=\lim_{x\to 7}\frac{(x-7)(x+7)}{(x-7)(x+2)}=\lim_{x\to 7}\frac{(x+7)}{(x+2)}=\frac{14}{9}$$

$$\lim_{x \to -2} \frac{x^2 - 4}{x^2 + 6x + 8}$$

$$\lim_{x \to -\frac{1}{2}} \frac{2x^2 + 7x + 3}{2x^2 - 3x - 2}$$

$$\lim_{h \to 0} \frac{(-5 + h)^2 - 25}{h}$$

$$\lim_{x \to 3} \frac{\frac{1}{x} - \frac{1}{3}}{x - 3}$$

$$\lim_{x \to 0} \left(\frac{1}{t} - \frac{1}{t^2 + t} \right)$$

Technique 3: Rationalization [2.4]

This technique is applied if we get an indeterminate form and the question contains a square root. By multiplying by the conjugate one can solve the problem. The following example illustrates the process.

$$\lim_{x\to 23}\frac{\sqrt{x+2}-5}{x-23}=\lim_{x\to 23}\frac{\sqrt{x+2}-5}{x-23}\times\frac{\sqrt{x+2}+5}{\sqrt{x+2}+5}=\lim_{x\to 23}\frac{x+2-25}{(x-23)(\sqrt{x+2}+5)}$$

$$=\lim_{x\to 23}\frac{x-23}{(x-23)(\sqrt{x+2}+5)}=\lim_{x\to 23}\frac{1}{\sqrt{x+2}+5}=\frac{1}{\sqrt{23+2}+5}=\frac{1}{10}$$

$$\lim_{x\to 0}\frac{\sqrt{9+x}-3}{x}$$

Technique 4: Absolute value [2.4]

Recall that the absolute value of a function forces the function to equal a positive value. Therefore if the value inside the absolute value is positive then the absolute value does nothing. However, if the value inside the absolute value is negative then it can be forced to be positive by multiplying by -1:

| 3 | = 3
| -3 | = -(-3) = +3
This is the idea that we exploit when solving absolute value problems:

$$\lim_{x \to -3^-} \frac{|x+3|}{x^2-9} = \lim_{x \to -3^-} \frac{-(x+3)}{(x+3)(x-3)} = \lim_{x \to -3^-} \frac{-1}{(x-3)} = \frac{-1}{-6} = \frac{1}{6}$$

$$\lim_{x \to -3^+} \frac{|x+3|}{x^2-9} = \lim_{x \to -3^+} \frac{(x+3)}{(x+3)(x-3)} = \lim_{x \to -3^+} \frac{1}{(x-3)} = \frac{1}{-6} = \frac{-1}{6}$$

$$\lim_{x \to -3} \frac{|x+3|}{x^2-9} = does\ not\ exist$$

$$\lim_{x \to -3} \frac{|x+3|}{x^2+7x+12}$$

Remaining topics to learn:

Limits at Infinity

Squeeze Theorem

Intermediate Value Theorem